Exercise 60

(a) Prove that the equation has at least one real root. (b) Use your graphing device to find the root correct to three decimal places.

$$\arctan x = 1 - x$$

Solution

Bring both terms to the same side.

$$\arctan x + x - 1 = 0$$

The function $f(x) = \arctan x + x - 1$ is continuous everywhere because it's the sum of two functions known to be continuous everywhere, the inverse tangent function and a polynomial function.

$$f(x) = 0$$

Find a value of x for which the function is negative, and find a value of x for which the function is positive.

$$f(0) = -1$$

$$f(1) \approx 0.785$$

$f(x)$ is continuous on the closed interval $[0, 1]$, and $N = 0$ lies between $f(0)$ and $f(1)$. By the Intermediate Value Theorem, then, there exists a root within $0 < x < 1$. Find other values of x within this interval for which the function is negative and positive.

$$f(0.5) \approx -0.0364$$

$$f(0.6) \approx 0.140$$

$f(x)$ is continuous on the closed interval $[0.5, 0.6]$, and $N = 0$ lies between $f(0.5)$ and $f(0.6)$. By the Intermediate Value Theorem, then, there exists a root within $0.5 < x < 0.6$. Find other values of x within this interval for which the function is negative and positive.

$$f(0.520) \approx -0.000480$$

$$f(0.521) \approx 0.00131$$

$f(x)$ is continuous on the closed interval $[0.520, 0.521]$, and $N = 0$ lies between $f(0.520)$ and $f(0.521)$. By the Intermediate Value Theorem, then, there exists a root within $0.520 < x < 0.521$. The function is closer to zero at $x = 0.520$ than it is at $x = 0.521$. Therefore, to three decimal places, the root is

$$x \approx 0.520.$$

This is reflected in the graph of $f(x)$ versus x.